Event-Triggered State Estimation, Control and Learning

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Abstract

Event-triggered sampling and control provides an effective solution to actively acquire measurement information and selectively perform estimation or control updates for enhanced system performance. In this chapter, we provide a concise overview of event-triggered state estimation and control developed for networked systems with limited communication or computation resources, as well as the recent developments on event-triggered learning that enable active discovery of system dynamics with enhanced data efficiency. Key technical challenges, major developments, and potential future directions are also discussed.

Key Points

- A holistic introduction of event-triggered sampled-data control systems is presented.
- State-of-the-art developments on event-triggered estimation and control are reviewed.
- Recent advances in event-triggered learning and learning-based control are introduced.

Introduction

With advancements in sensing technology and emerging applications of advanced control, how to efficiently acquire and make use of data has become one of the most important questions to answer in control systems design. Event-triggered sampling and data management protocols offer answers to this question, which have attracted lots of research attention during the past two decades.

Early investigations focused on state estimation and control problems based on event-triggered sampling, with the aim of overcoming the potential over-sampling in traditional periodic sampling and control protocols (Åström and Kumar, 2014). Such

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problems need to be considered when the communication and computation resources are limited in a control system (e.g., wireless networked control systems powered by batteries). In particular, when the control system is in steady state, periodic sampling may result in excessive use of communication resources, as no sensor updates need to be reported or no control actions need to be taken. In contrast, for event-triggered sampling protocols, the targeted system variables are sampled or transmitted only when pre-specified event-triggering conditions are satisfied. The key challenge in event-triggered state estimation and control is how to maintain estimation and control performance with reduced average sampling rates by exploiting the structural properties of the underlying event-triggering conditions. The pioneering work on optimal event-triggered control for first-order stochastic systems (Åström and Bernhardsson, 2002) showed the potential of event-triggered sampling in improving control performance under the same average sampling rate. Optimal sampling problems were studied in time-stopping formulations (Rabi *et al.*, 2006, 2012). Later the stability issues of event-triggered control were investigated using Lyapunov approaches (Donkers and Heemels, 2012; Heemels and Donkers, 2013; Tabuada, 2007). Event-triggered versions of many standard problems considered in time-triggered control were then studied, e.g., optimal state estimation (Shi *et al.*, 2014; Sijs and Lazar, 2012; Wu *et al.*, 2013), model predictive control (Eqtami *et al.*, 2011; Li and Shi, 2014), and distributed control and coordination of multi-agent systems (Dimarogonas *et al.*, 2012). Systematic overviews of the theoretic developments can be found in Heemels *et al.* (2012) and Shi *et al.* (2016b). Applications of event-triggered control and estimation were also reported for, e.g., biomedical engineering systems (Chakrabarty *et al.*, 2018), aerospace systems (Li *et al.*, 2022), process control (Ahlen *et al.*, 2019), and robotics (Ding *et al.*, 2023).

Motivated by the availability of sensor data and the propelling performance requirements of intelligent autonomous systems, there has been a recent surge in the study of data-driven learning-based control. With the abundance of data and the lack of plant models, a natural question to ask is when to learn and how to learn with guaranteed performance and data efficiency. Event-triggered learning solves this problem by evaluating the relevance of the data online and only performing learning updates when the data samples are relevant and important for control performance. Early theoretic results were reported in Solowjow and Trimpe (2020a) and Umlauft and Hirche (2020a), and application of event-triggered learning in robotics and mobile health can be found in Shi *et al.* (2019) and Umlauft *et al.* (2020).

In this chapter, we provide an overview of event-triggered state estimation, control, and learning. We begin with a brief introduction of event-triggering conditions in Section "Event-Triggering Conditions". Results on event-triggered estimation and control are presented in Section "Event-Triggered State Estimation" and Section "Event-Triggered Control", respectively, and an introduction of event-triggered learning is given in Section "Event-Triggered Learning". Concluding remarks are provided in Section "Summary".

Event-Triggering Conditions

In control systems, event-triggered strategies are implemented to reduce the frequency of message exchanges among sensors, controllers, and actuators. These strategies utilize specific conditions, encoded through a triggering function denoted as $\Psi(\cdot)$, to determine when new data should be transmitted based on a given state and error combination. Various triggering conditions have been proposed in the literature, including those based on estimation error in Xia *et al.* (2016), predicted output error like (Trimpe, 2014), functions of the estimation error by Han *et al.* (2015) and Wu *et al.* (2013), or error covariance in Trimpe and D'Andrea (2014).

The selection of event-triggering conditions should be based on the specific requirements of the control system, the characteristics of the controlled plant, and the communication infrastructure. Each type of condition offers distinct advantages and can significantly impact the overall performance, efficiency, and reliability. Furthermore, it is possible to combine these conditions in the design of event-triggering laws. Below we give a set of the most common event-triggering conditions in the literature.

Static Event-Triggering Conditions

A static event-triggering condition depends solely on the current system state at each time instant t. Once the value of the triggering function surpasses a predetermined threshold, an event is triggered, and new information is transmitted to the controller. Suppose event-triggered data transmission occurs at $\{t_0, t_1, \ldots, t_k, \ldots\}$. The general expression of static event-triggering conditions is given by:

$$t_{k+1} := inf\,\{t > t_k | \Psi(\cdot) > 0\}. \tag{1}$$

If $\Psi(\cdot)$ is a deterministic function, condition (1) corresponds to the deterministic event-triggered mechanism, as proposed in Wu *et al.* (2013). On the other hand, if $\Psi(\cdot)$ is a stochastic function dependent on a random variable, (1) transforms into a stochastic event-triggering law, such as the one developed in Han *et al.* (2015). While static event-triggering conditions are easy to implement and understand, they do not adapt to the system's current state or performance. Consequently, they may lead to conservative behavior and fail to account for changes in system dynamics or the environment.

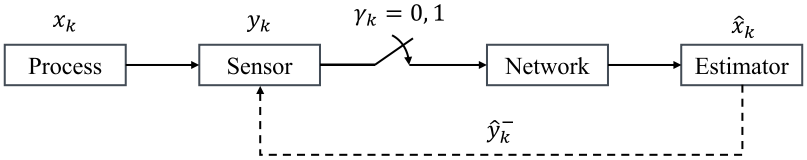

Fig. 1 Event-triggered sensor scheduling diagram for remote state estimation.

Dynamic Event-Triggering Conditions

The dynamic event-triggered mechanism introduces dynamic variables into the event-triggering condition, allowing for additional flexibility in deciding whether to transmit measurements at each time instant. This dynamic condition considers not only the current measurement but also the previous triggering state at last triggering instant. The general expression of dynamic event-triggering conditions can be described as:

$$t_{k+1} := inf\{t > t_k | \Psi(\cdot) > \chi(t)\},$$

where χ is a dynamic variable based on the system state and error estimates. For example, Huang *et al.* (2016) developed a dynamic event-trigger for hidden Markov models, which depends on sensor measurements and previous triggering states. Yi *et al.* (2018) proposed an event-triggered mechanism for multi-agent consensus. Compared to static event-triggering, dynamic event-triggering conditions are more complex to design and implement. Moreover, they often require triggering state feedback to tune the triggering mechanism.

Time-Regularized Event-Triggering Conditions

In continuous-time systems, time-regularized event-triggering conditions combine time-based elements with event-based criteria. In these conditions, transmissions are triggered not only by errors exceeding a threshold, but also occur at regular time intervals enforced by a minimum inter-event time (MIET), τ_{MIET}, which guarantees a finite number of events in finite time and thus prevents Zeno behavior. A typical expression for time-regularized event-triggering conditions can be written as:

$$t_{k+1} := inf\{t > t_k + \tau_{\mathrm{MIET}} | \Psi(\cdot) > 0\}.$$

Time-regularized triggering methods (Dolk *et al.*, 2014, 2016) are proposed to ensure L_p-stability of nonlinear systems. These methods involve checking the triggering condition only after a specific amount of time has passed since the most recent triggering instant.

Periodic Event-Triggering Conditions

In addition to time-regularized event-triggering conditions, periodic event-triggering conditions are also proposed, for which the event-triggering condition is only evaluated periodically. Suppose that the sampling period is $h > 0$; a typical periodic event-triggering condition is:

$$t_{k+1} := \inf_{i \in \mathbb{N}} \{ih > t_k | \Psi(\cdot) > 0\}.$$

Event-Triggered State Estimation

Remote state estimation aims to accurately estimate a system's state using sensor data transmitted to a remote estimator. This setup allows the estimator to make informed decisions or provide feedback to the system. By incorporating event-triggered sensor scheduling into remote state estimation, communication frequency can be reduced, thereby prolonging the operational lifespan of battery-sensor devices. **Fig. 1** illustrates the process of remote state estimation with an event-triggered mechanism. Here, $x_k \in \mathbb{R}^n$ represents the state vector, $y_k \in \mathbb{R}^m$ the sensor measurement, $\hat{x}_k$ the state estimate, $\hat{y}_k^-$ potential feedback, and γ_k the event-triggered scheduler at time k. Consider the linear system

$$x_{k+1} = Ax_k + w_k, \tag{2}$$

$$y_k = Cx_k + v_k, \tag{3}$$

where $w_k \in \mathbb{R}^n$ and $v_k \in \mathbb{R}^m$ are mutually uncorrelated white Gaussian noises with covariance matrices $Q \succ 0$ and $R \succ 0$, respectively, where the notation indicates that the matrices are positive definite. The initial state x_0 is zero-mean Gaussian with covariance matrix Σ_0, and is uncorrelated with w_k and v_k for all $k \geq 0$. It is common to assume that (A, C) is detectable. In the absence of an event-triggered mechanism, the state can be estimated using a Kalman filter (Kailath *et al.*, 2000). The update procedure is as follows:

Time Update:

$$\hat{x}_k^- = A\hat{x}_{k-1}^-, \quad P_k^- = AP_{k-1}A^T + Q,$$

Measurement Update:

$$K_k = P_k^- C^T [CP_k^- C^T + R]^{-1},$$

$$\hat{x}_k = \hat{x}_k^- + K_k[y_k - C\hat{x}_k^{-1}], \quad P_k = P_k^- - P_k^- C^T [CP_k^- C^T + R]^{-1} CP_k^{-1},$$

where the state estimate $\hat{x}_k \in \mathbb{R}^n$ is optimal in the sense of minimum mean square error (MMSE). As $k \to \infty$, the Kalman filter converges, i.e., $K_k \to K$, $P_k^- \to \bar{P}$, and $P_k \to P$, where K, $\bar{P}$>0 and P>0 are constant matrices. It can be shown that the residues $y_k - C\hat{x}_k^-$ are independently and identically distributed Gaussian process with zero mean and covariance $P := C\bar{P}C^T + R$ Anderson and Moore (1979). This result plays an important role in analyzing the performance of different event-triggering laws.

Estimation With Reliable Communication Channels

When transmitted data are accurately received by the remote estimator, several event-triggered state estimates have been proposed. Sijs and Lazar (2012) developed an event-based estimator based on a general description of event-based sampling. This approach approximates the uniform distribution with a finite sum of Gaussian distributions. Wu *et al.* (2013) designed a deterministic event-triggered scheduler defined by

$$\gamma_k = \begin{cases} 0 & \text{if } ||\varepsilon_k||_\infty \leq \delta, \\ 1 & \text{otherwise}, \end{cases} \tag{4}$$

where δ represents a predefined threshold, and ε_k is the normalized innovation vector. As the conditional distribution of the system state x_k, given the information $I_k \triangleq \{\gamma_0, \ldots, \gamma_k, \gamma_0 y_0, \ldots, \gamma_k y_k\}$, becomes truncated Gaussian under this event-triggered mechanism, the exact MMSE estimator involves complex numerical integration. Therefore, based on a Gaussian approximation assumption, they derived an approximate estimator, which includes the following steps:

Time Update:

$$\hat{x}_k^- = A\hat{x}_{k-1}^-, \quad P_k^- = AP_{k-1}A^T + Q. \tag{5}$$

Measurement Update:

$$\hat{x}_k = \hat{x}_k^- + \gamma_k(P_k^- C^T [CP_k^- C^T + R]^{-1}) z_k, \quad P_k = P_k^- - [\gamma_k + (1-\gamma_k)\beta(\delta)] P_k^- C^T (CP_k C^T + R)^{-1} CP_k^-, \tag{6}$$

where

$$z_k = y_k - \hat{y}_k^-, \quad \beta(\delta) = \frac{2}{\sqrt{2\pi}} \delta e^{-\frac{\delta^2}{2}} [1 - 2Q(\delta)]^{-1}, \quad Q(\delta) = \int_\delta^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx.$$

The average sensor communication rate is defined as $\gamma \triangleq limsup_{T\to\infty} \frac{1}{T+1} \sum_{k=0}^{T} \mathbb{E}[\gamma_k]$. Under the event-triggered scheduler (4) and the estimator (5–6), γ=1 − $[1 - 2Q(\delta)]^m$. The corresponding results for more general event-based data schedulers and multiple sensors were proposed in Shi *et al.* (2014).

To preserve the Gaussian property and obtain the exact MMSE estimates in recursive and closed form, Han *et al.* (2015) proposed the stochastic event-triggered sensor schedulers

$$\gamma_k = \begin{cases} 0, & \zeta_k \leq \phi(y_k, \hat{y}_k^-), \\ 1, & \zeta_k > \phi(y_k, \hat{y}_k^-), \end{cases} \tag{7}$$

where ζ_k is uniformly distributed over the interval [0,1]. By restricting the range of ϕ to {0,1}, the stochastic event-trigger becomes deterministic as in (4).

- Open-Loop Schedule Function: When the system is stable, a remote estimator can utilize the open-loop schedule function without feedback $\hat{y}_k^-$ to the sensor. In this case

$$\phi(y_k, \hat{y}_k^-) \triangleq exp\left(-\frac{1}{2}y_k^T Y y_k\right), \tag{8}$$

where $Y > 0$ is a matrix parameter. Under such event-trigger, the system state x_k conditioned on I_{k-1} remains Gaussian. Consequently, the exact MMSE estimator can be derived as follows:

Time Update:

$$\hat{x}_k^- = A\hat{x}_{k-1}^-, \quad P_k^- = AP_{k-1}A^T + Q,$$

Measurement Update:

$$\hat{x}_k = (I - K_k C)\hat{x}_k^- + \gamma_k K_k y_k, \quad P_k = P_k^- - K_k C P_k^-,$$

where $K_k = P_k^- C^T [CP_k^- C^T + R + (1-\gamma_k)Y^{-1}]^{-1}$. When A is stable, the average communication rate $\gamma = lim_{N\to\infty}\frac{1}{N}\sum_{k=0}^{N-1}\gamma_k = 1 - \frac{1}{\sqrt{det(I + \Pi Y)}}$, where Π is the covariance of the state x_k at steady state, and Y is the matrix parameter in (8).

- Closed-Loop Schedule Function: If the sensor can receive feedback $\hat{y}_k^-$ from the estimator before making a decision, it can compute the innovation z_k and use the closed-loop event-trigger

$$\phi(y_k, \hat{y}_k^-) \triangleq exp\left(-\frac{1}{2}(y_k - \hat{y}_k^-)^T Z (y_k - \hat{y}_k^-)\right), \tag{9}$$

where $Z > 0$ is a matrix parameter. Then, the remote estimator is derived as follows:

Time Update:

$$\hat{x}_k^- = A\hat{x}_{k-1}^-, \quad P_k^- = AP_{k-1}A^T + Q,$$

Measurement Update:

$$\hat{x}_k = \hat{x}_k^- + \gamma_k K_k z_k, \quad P_k = P_k^- - K_k C P_k^-,$$

where $K_k = P_k^- C^T [CP_k^- C^T + R + (1-\gamma_k)Z^{-1}]^{-1}$.

The open-loop scheduler is simpler to implement as it does not require feedback. However, it cannot reduce the communication rate for unstable systems since $\gamma=1$ almost surely for any given Y. Closed-loop schedulers are necessary to decrease the communication rate. Additionally, the design matrices Y or Z provide extra flexibility to balance the trade-off between communication rate and the performance.

Extensions to multi-sensor stochastic event-triggered estimation were explored by Weerakkody *et al.* (2015). Huang *et al.* (2017) studied event-triggered state estimation for linear Gaussian systems with energy harvesting sensors, obtaining recursive MMSE estimates for both state and sensor energy levels. To find the optimal trade-off between MMSE and communication costs, Molin and Hirche (2017) treated the joint optimal design of estimator and event-trigger as a two-person game and devised an iterative scheme involving dynamic programming and conditional expectation. Under the open-loop event-triggered mechanism proposed in Mohammadi and Plataniotis (2017), the triggering condition is independent of local state estimates, and the estimator does not require prior information about the sensors' triggering mechanism. These advantages simplify the implementation. Moreover, Petri *et al.* (2021) introduced a dynamic event-triggered Luenberger observer to ensure global asymptotic stability for estimation error dynamics. Weimer *et al.* (2012) explored distributed event-triggered estimation in networked systems, proposing a heuristic one-step greedy algorithm to minimize a weighted function of network energy consumption and transmission counts subject to estimator performance constraints. To deploy sensors over a peer-to-peer network, Battistelli *et al.* (2018) developed a consensus Kalman filter with event-triggered communication, guaranteeing MMSE boundedness.

Estimation With Lossy Channels

Packet drops are often inevitable using wireless communication and can have a significant impact on event-based sensor scheduling. When packet dropouts occur, the remote estimator faces the challenge of distinguishing between packet loss due to poor channel conditions and loss of triggering events. If a sensor measurement is below a given threshold, this information can be used to enhance the estimate. However, when caused by channel loss, the estimator lacks information about the sensor measurement and cannot perform an update. This complication adds difficulty to the design of optimal estimators. Many researchers have dealt with event-triggered state estimation with lossy communication. Kung *et al.* (2020) demonstrated that the Gaussian property cannot be preserved even under stochastic event-trigger. In studies of the effect of a lossy channel on event-based estimation, Shi *et al.* (2016a) utilized the reference probability measure approach to leverage event-triggered measurement information. Xu *et al.* (2020) proved that when the estimator only knows the channel loss rate, the system state conditioned on available information follows a Gaussian mixture distribution. To address the challenges associated with optimal MMSE, they developed estimators with reduced computation and memory complexity by minimizing the length and number of hypotheses. Different from these works, assuming independently and identically distributed packet drops, Chen *et al.* (2017) investigated a generic stochastic event-triggering law under the Gilbert-Elliott communication channel. In some vulnerable channels, packet drops may result from denial of service attacks. Li *et al.* (2021) investigated stochastic event-triggered distributed fusion estimation under jamming attacks, establishing a Stackelberg game framework to analyze interactions between attackers and smart sensors. Huang *et al.* (2021) explored the event-triggered scheduling problem in cognitive radio sensor networks, where the uncertainty from the event trigger and communication channels can be decoupled. Furthermore, Zhong *et al.* (2023) proposed a novel formulation incorporating error-detecting codes and developed approximate MMSE estimators that achieve nearly the same error with significantly lower computation time. In multi-agent systems, Bemani and Björsell (2021) designed a distributed event-triggered state estimation method while maintaining satisfactory control performance even under high probabilities of packet drops. For nonlinear discrete systems, event-based state estimation under packet drops was studied by Gasmi *et al.* (2024) using particle filters.

Event-Triggered Control

In event-triggered control, the execution of controllers is driven by an event-triggering condition. Since the controller is only executed when needed, event-triggered control can effectively reduce resource consumption while addressing various control objectives such as stabilization (Eqtami *et al.*, 2010; Heemels *et al.*, 2012; Tabuada, 2007; Xing *et al.*, 2017; Yu *et al.*, 2019), reference tracking (Vamvoudakis *et al.*, 2017; Zhang *et al.*, 2020), output regulation (Liu and Huang, 2017), and consensus of multi-agent systems (Dimarogonas *et al.*, 2012; Ge and Han, 2017; He *et al.*, 2017; Zhang *et al.*, 2020). The main issues considered in event-triggered control include how to design the controller and the compatible triggering condition such that a systematic trade-off between control performance and transmission efficiency can be obtained. Generally, the design methods can be classified based on the design sequence of controllers and event-triggering conditions, and thus can be categorized as emulation-based approaches and co-design approaches. In this section, we will take the stabilization problem as an example to illustrate the design of controllers and triggering conditions.

Emulation-Based Approaches

Emulation-based approaches investigate the design of controller and triggering condition separately through a two-step process. First, a stable controller is designed on the basis of traditional model-based control theory. Then with this stable controller, a compatible triggering condition is designed to save resources without violating the stability of the closed-loop system.

Linear plants

Considering the linear system

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{10}$$

where $x(t) \in \mathbb{R}^n$ is the system state and $u(t) \in \mathbb{R}^m$ the control input. Assuming the stabilizability of (A, B), a state-feedback control law $u(t) = Kx(t_k)$, where $K \in \mathbb{R}^{m \times n}$, can be designed to ensure the stability of $\dot{x}(t) = (A + BK)x(t)$.

- Continuous-time Event-triggered Control: In early studies of event-triggered control, the system is evaluated continuously and the control task is executed when event-triggering condition is satisfied. Taking the event-triggering condition based on the state error as an example, the triggering instant t_{k+1} is determined by

$$t_{k+1} = inf\{\ t > t_k \ \mid \ |e(t)| \geq \sigma |x(t)| \ \}, \tag{11}$$

where $|\cdot|$ denotes Euclidean norm, $e(t) := x(t_k) - x(t)$, and $k \in \mathbb{N}$. To prevent Zeno behavior, the triggering condition is designed to ensure a positive MIET, which can be analyzed by investigating the underlying closed-loop dynamics and the triggering condition. It is proved to exist in Tabuada (2007). However, when it comes to a more complex system, a continuous-time triggering scheme may not guarantee a positive MIET. To mitigate this issue, advanced event-triggered control schemes were developed for systems with external disturbances (Heemels and Donkers, 2013; Heemels *et al.*, 2012, 2013), different control objectives (Dolk *et al.*, 2016; Vamvoudakis *et al.*, 2017; Zhang *et al.*, 2020), output-feedback (Donkers and Heemels, 2012; Zhang and Han, 2014), and distributed configurations (Dimarogonas *et al.*, 2012; Ge and Han, 2017; He *et al.*, 2017; Yi *et al.*, 2018).

- Periodic Event-triggered Control: To alleviate the requirement of high-frequency monitoring during the implementation of continuous-time event-triggered control schemes, periodic event-triggered control approaches were introduced (Heemels *et al.*, 2012, 2013), where the event-triggering condition is only evaluated periodically, thus enabling sampled-data control implementation. Suppose that the sampling period is $h > 0$, then the triggering instant can be described as $t_{k+1}h$, which is determined by

$$t_{k+1} = \inf_{i \in \mathbb{N}} \{ i > t_k \,|\, |e(ih)| \geq \sigma |x(ih)| \}. \tag{12}$$

- Perturbed System Approach: The triggering errors (the difference between the current and previous transmitted signals) can be characterized as bounded perturbations, allowing analysis of a perturbed system (Eqtami *et al.*, 2010; Mazo and Tabuada, 2011; Tabuada, 2007). While this approach simplifies the analysis, it neglects the underlying error dynamics to a certain extent.
- Hybrid System Approach: To offer an insight into the inter-event behaviors, the continuous-time system dynamics (in continuous time) can be categorized and investigated according to the triggering status and is modeled as a hybrid system (Borgers and Heemels, 2014; Donkers and Heemels, 2012; Yu and Chen, 2021).
- Piecewise System Approach: As a discretized analogue of the hybrid system approach, discrete-time event-triggered control systems can be modeled as discrete-time bimodal systems according to the triggering status (Heemels and Donkers, 2013).

Nonlinear plants

For nonlinear system, most existing studies are based on input-to-state stability (ISS) (Liu and Jiang, 2015; Tabuada, 2007; Zhang *et al.*, 2017). Consider the following nonlinear system:

$$\dot{x}(t) = f(x(t), u(t)), \quad x(t) \in \mathbb{R}^n, \quad u(t) \in \mathbb{R}^m \tag{13}$$

with feedback controller $u(t) = k(x(t))$. The controller is designed to render the closed-loop system $\dot{x}(t) = f(x(t), k(x(t) + e(t)))$ to be ISS with respect to the measurement error $e(t) \in \mathbb{R}^n$. Given the controller $u(t) = k(x(t))$ such that the ISS of the closed-loop system is satisfied, the even-triggering condition can be designed. For instance, the triggering condition can be chosen such that $\gamma(|e|) \geq \sigma\alpha(|x|)$, $\sigma \in (0,1)$ (Tabuada, 2007), which ensures $\frac{\partial V}{\partial x} f(x, k(x+e)) \leq (\sigma - 1)\alpha(|x|) \leq 0$. However, the assumption of the closed-system being ISS with respect to measurement errors may become restrictive for some nonlinear systems. To mitigate this issue, several studies utilized the concept of integral ISS (iISS) (Mousavi *et al.*, 2019; Yu *et al.*, 2019). However, with this looser restriction on the predefined controller, a compatible triggering condition could only be designed for a specific set of initial states of the system. For instance, to ensure the ultimately bounded stabilization of the closed-loop system, the triggering condition is designed as $\gamma(|e|) > c_0$ in Yu *et al.* (2019), where the design of c_0 is related to the constraint of the initial state. In addition, periodic event-triggering schemes have been extended to nonlinear systems (Postoyan *et al.*, 2015; Wang *et al.*, 2020b).

Co-Design Approaches

Although the emulation-based approached simplify the design of the controller and the triggering condition, it is a bit difficult to obtain an optimal trade-off between the control performance and the resource utilization, since the design of triggering conditions relates closely to the dynamics of the closed-loop system determined by the predefined controller. To address this issue, co-design approaches were proposed in recent studies.

LMI-based methods

For linear systems, a Lyapunov-based stability condition or a condition for certain performance specification can be established by integrating the system dynamics and the triggering condition. This condition is then reformulated into a linear matrix inequality (LMI). For instance, when a state-feedback stabilization problem is considered under the triggering condition defined by σ in (12), the LMI can be formulated as the positive semi-definite matrix condition $M(K, \sigma) \pm 0$, where K is the feedback gain, σ is the parameter of the triggering condition. Note that a large σ leads to less frequent transmissions. The optimal σ to reduce the transmission frequency can be designed by solving

$$\begin{aligned} &\max \ \sigma \\ &\text{subject to } \ M(K, \sigma) \geq 0. \end{aligned}$$

Leveraging LMI-based co-design approaches, stabilization problems under scenarios such as output feedback control (Zhang and Han, 2014), robust control (Peng and Han, 2013), saturated systems (Seuret *et al.*, 2016), and time-delay systems (Yue *et al.*, 2013) were also considered.

Backstepping-based methods

In the emulation-based design of nonlinear systems, stability conditions such as ISS and iISS need to be guaranteed under the design of the triggering condition. However, one might not be able to find a controller such that the closed-loop system is ISS or iISS. To address this issue, the backstepping technique is utilized to achieve event-triggered control of strict-feedback nonlinear systems in Li and Yang (2018), Wang and Krstic (2022), Wang *et al.* (2020a) and Xing *et al.* (2017), where virtual inputs were derived recursively and then modified based on the event-triggering conditions. For example, in Xing *et al.* (2017), the proposed event-triggered controller and the corresponding triggering condition were formulated as

$$w(t) = v(t) - f(\bar{m})$$

$$u(t) = w(t_k),\ t \in [t_k,\ t_{k+1})$$

$$t_{k+1} = inf\{\ t \in \mathbb{R}\ \mid\ |w(t) - u(t)| \geq m\ \},$$

where $v(t_k)$ is the virtual input computed by the backstepping method, and $\bar{m}$ and m are parameters that need to be chosen. Backstepping allows the co-design of the controller and the triggering without necessitating a predefined ISS or iISS controller or the global Lipschitz continuity of the considered system.

Event-Triggered Learning

Beyond event-triggered state estimation and control, there has been an increasing attention on event-triggered learning in recent years. Event-triggered learning methods learn from partial selected data and learn only when necessary. The existing approaches can be categorized into non-parametric and parametric approaches.

Non-Parametric Event-Triggered Learning

Non-parametric event-triggered learning mechanisms were motivated by improving data efficiency through active training data selection.

Consider a nonlinear function $f_{true}\colon\ \mathbb{X} \to \mathbb{R}$ and a relation

$$y = f_{true}(\mathbf{x}) + \varepsilon, \tag{14}$$

where ε is the noise, $\mathbf{x} \in \mathbb{X} \subseteq \mathbb{R}^n$ the input, and $y \in \mathbb{R}$ the output. Several event-triggered learning methods were proposed for such a setup.

Using Gaussian process regression, Umlauft and Hirche (2020b) proposed an event-triggered online learning method for Gaussian noise $\varepsilon\colon \mathbf{N}(0,\sigma_\varepsilon^2)$. The unknown function $f_{true}(\cdot)$ was learned as a distribution over functions (Seeger, 2004), denoted by

$$f_{GP}(\mathbf{x})\pi \sim GP(m(\mathbf{x}),\, k(\mathbf{x},\, \mathbf{x}')). \tag{15}$$

The learned distribution is fully specified by a mean $m(\mathbf{x})\colon \mathbb{X} \to \mathbb{R}$ and a kernel function $k(\mathbf{x}, \mathbf{x}')\colon \mathbb{X} \times \mathbb{X} \to \mathbb{R}$, and trained using a time-varying dataset $\mathbb{D}_k$. In the proposed method, the dataset is updated at time k_h and remains constant until k_{h+1}, and $N_k \in \mathbf{N}$ denotes the current number of data points. Specifically, the updating time k_{h+1} is determined by a static event-triggering condition in the form of (1) with $\Psi_k(\mathbf{x})$ being

$$\Psi_k(\mathbf{x}) := min\left\{\beta_k\sigma_k(\mathbf{x}) - k_c|r|,\ ||\mathbf{e}|| - \frac{\sigma_\varepsilon\beta_k}{k_c\|[\boldsymbol{\lambda}^{\mathrm{T}}1]\|}\right\}, \tag{16}$$

where $\beta_k = \sqrt{2B_f + 300\gamma_k log^3\left(\frac{k+1}{\delta}\right)}$, $\sigma_k(\cdot)$ is the posterior variance function of the Gaussian process with training dataset $\mathbb{D}_k$, $k_c \in \mathbb{R}_+$ is a constant, $\mathbf{e}$ is the tracking error, B_f is the upper bound of the reproducing kernel Hilbert space norm of function $f_{true}(\cdot)$, and $r = [\boldsymbol{\lambda}^{\mathrm{T}}\ 1]\mathbf{e}$ is a filtered scalar state with coefficient vector $\boldsymbol{\lambda}$. Moreover, Umlauft and Hirche (2020b) designed a feedback linearization controller for the nonlinear system. The global uniform ultimate boundedness of the tracking error was proved for the proposed even-triggered learning method.

Based on the lazily adapted constant kinky inference approach introduced in Calliess *et al.* (2020), Zheng *et al.* (2023), proposed two kinds of event-triggered learning methods. Using the dataset $\mathbb{D}_k = \left\{(\mathbf{x}_{k_i}, y_{k_i})\right\}_{i=1}^{N_k}$, the function in (14) is learned as

$$\hat{f}_k(\mathbf{x};\, L(k),\, \mathbb{D}_k) := \frac{1}{2}\mathfrak{u}_k(\mathbf{x};\, L(k)) + \frac{1}{2}l_k(\mathbf{x};\, L(k)),$$

$$\mathfrak{u}_k(\mathbf{x};\, L(k)) := \min_{i\in\{1,\ldots,N_k\}} y_{k_i} + L(k)||\mathbf{x} - \mathbf{x}_{k_i}||_\infty + \bar{e},$$

$$L_k(\mathbf{x};\, L(k)) := \max_{i\in\{1,\ldots,N_k\}} y_{k_i} - L(k)||\mathbf{x} - \mathbf{x}_{k_i}||_\infty - \bar{e},$$

where $\bar{e}$ is the upper bound of the noise such that $||\varepsilon||_\infty \leq \bar{e}$, and $L(k)$ is the learned Lipschitz constant. The learned parameter $L(k)$ is updated intermittently according to

$$L(k+1) := max\left\{L(k), \max_{(\mathbf{x}_{k_i}, y_{k_i}) \in \mathbb{D}_k} \frac{|y_{k_i} - y_{k_{i+1}}| - \lambda}{||\mathbf{x}_{k_i} - \mathbf{x}_{k_{i+1}}||_\infty}\right\} \tag{17}$$

with λ being a learning hyper-parameter. Then two event-triggering conditions were designed, namely, sample-grid based condition and prediction-error based. We note that the event-triggered mechanisms are all static conditions in the form of (1). Prediction errors were analyzed in Zheng *et al.* (2023) with the dataset $\mathbb{D}_k$ updated using the proposed event-triggered mechanisms. The specific triggering conditions and the corresponding learning error bounds are listed as follows, where L_{true} is the Lipschitz constant of $f_{true}(\cdot)$ in X.

- Sample-grid based event-triggered learning:

$$\Psi_k(\mathbf{x}_k) = dist(\mathbb{D}_{k_h}, \mathbf{x}_k) - \delta_1 := \min_{(\mathbf{x}_{k_i}, y_{k_i}) \in \mathbb{D}_{k_h}} ||\mathbf{x}_{k_i} - \mathbf{x}_k||_\infty - \delta_1. \tag{18}$$

Prediction error bound:

$$|\hat{f}_k(\mathbf{x}; L(k), \mathbb{D}_k) - f_{true}(\mathbf{x})| \leq (L(n) + L_{true})\delta_1 + \frac{\lambda}{2} + \bar{e}, \forall\, \mathbf{x} \in \mathbb{X}. \tag{19}$$

- Prediction-error based event-triggered learning:

$$\Psi_k(\mathbf{x}) = b_k(\mathbf{x}; L(n)) - \delta_2 := \frac{1}{2}|u_k(\mathbf{x}; L(k)) - l_k(\mathbf{x}; L(k))| - \delta_2. \tag{20}$$

Prediction error bound:

$$|\hat{f}_k(\mathbf{x}; L(k), \mathbb{D}_k) - f_{true}(\mathbf{x})| \leq \frac{L(n) + L_{true}}{L(n)}(\delta_2 - \bar{e}) + 2\bar{e}, \forall\, \mathbf{x} \in \mathbb{X}. \tag{21}$$

Other event-triggered learning methods were also proposed. For instance, an event-triggered nonlinear iterative learning control was presented by Lin *et al.* (2021), which was developed for repetitive nonaffine systems. Poveda and Teel (2017) proposed a learning framework for continuous-time nonlinear systems, where the dataset was updated according to an event-based update rule.

Parametric Event-Triggered Learning

For parametric models, event-triggered learning methods were proposed to deal with the mismatch between the learned model and the true dynamics. Specifically, a new model is identified if a certain metric of the system deviates significantly from the learned model. Solowjow and Trimpe (2020b) proposed event-triggered learning methods for linear time-invariant systems, where the mismatch is evaluated using inter-communication times of an event-triggered communication channel. Consider

$$\mathbf{x}_{k+1} = A\mathbf{x}_k + \varepsilon_k \tag{22}$$

with parameter matrix A and Gaussian noise $\varepsilon_k \sim \mathcal{N}(0, Q_\varepsilon^2)$, where $Q_\varepsilon^2 > 0$ is the covariance matrix. The learned model is denoted as $\hat{\mathbf{x}}_{k+1} = \hat{A}\hat{\mathbf{x}}_k$, where $\hat{\mathbf{x}}$ is the predicted state, and $\hat{A}$ is the identified parameter. The unknown system parameters are collected as $\theta = (A, Q_\varepsilon)$, while the learned parameters are $\hat{\theta} = (\hat{A}, \hat{Q})$. Using the event-triggering time instants $\{k_0, k_1, ..., k_{N_k}\}$, the inter-communication times $\boldsymbol{\tau}_k = \{\tau_0, \tau_1, ..., \tau_{N_k - 1}\}$ can be calculated as $\tau_i = k_{i+1} - k_i$, $i \in \{0, 1, ...\}$. Using the upper bound of the inter-communication time τ_{max}, two kinds of event-triggering conditions were designed, namely, expectation-based learning trigger and density-based learning trigger. We note that the learning triggers were designed using inter-communication times rather than previous triggering states. Thus the event-triggering conditions in Solowjow and Trimpe (2020b) are static event-triggering conditions in the form of (1), which were designed as follows.

- Expectation-based event-triggered learning:

$$\Psi_k(\boldsymbol{\tau}_k) = \left|\frac{1}{N_k - 1}\sum_{i=1}^{N_k - 1}\tau_i - \tau_i[\tau]\right| - \tau_{max}\sqrt{\frac{1}{2(N_k - 1)}ln\frac{2}{\alpha}}. \tag{23}$$

- Density-based event-triggered learning:

$$\Psi_k(\tau_k) = \sup_{t \in \mathbb{N}} |F(t) - F_k(t)| - \sqrt{\frac{1}{2(N_k - 1)} ln \frac{2}{\alpha}}, \tag{24}$$

where $F_k(t)$ is the cumulative distribution function obtained from set τ_k, and $F(t)$ is the cumulative distribution function of variable τ_i. In practice, the expectation $\mathbb{E}[\tau]$ and the cumulative distribution function $F(t)$ are unknown. Monte Carlo simulation approaches were adopted to estimate $\mathbb{E}[\tau]$ and $F(t)$. Other results and applications of parametric event-triggered learning were also provided by Baumann *et al.* (2019), Schlüter *et al.* (2020) and Solowjow *et al.* (2018).

Summary

In this chapter, we summarized the key ideas and developments in event-triggered control, state estimation and learning. Despite an extensive literature, a number of important problems need to be further studied, and the theoretic understanding of event-triggered learning is still at an early age. In particular, how to make efficient use of resources to actively acquire online measurements and develop systematic event-triggered learning for control approaches is yet to be largely explored.

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